With a decrease in the initial flow rate q_1, the boundary stream formed in the case of discharge from a slit nozzle with a dumbbell shape becomes more and more like the boundary stream formed on the edge of a nozzle in the form of a highly prolate ellipse, when $q_1 = 0$ and initial conditions (2.4) are valid.

Finally, let us examine data pertaining to fairly long films bounded by three free rims. Figure 5 shows in dimensionless form the results of calculation of discharge from a slit nozzle with a dumbbell-shaped edge (We = 10^3, Re = $0.28 \cdot 10^3$, K = 10^{-4}, $q_1 = 0.1$, $r_1 = 5$, $\theta_0 = 60°$). Curve 1 is for one of two free rims beginning on the nozzle edge ("oblique discontinuity), curve 2 is for a free rim with an axis in the form of a circle arc ("normal discontinuity"), and curve 3 is the distribution of the azimuthal stress $\sigma_{\theta\theta}$ along the film. The maximum of this stress and the subsequent sharp reduction are due to relaxation. For the parameter values corresponding to Fig. 4, the film becomes longer and a third rim develops behind it as a result of an increase in the divergence angle θ_0.

It can be suggested that the appearance of a third free "unloading rim" in the formal solution before joining of the boundary streams corresponds physically to breakdown of the film.

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NONSYMMETRIC COLLISION OF PLANE JETS OF AN IDEAL INCOMPRESSIBLE FLUID

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The problem of the nonsymmetric collision of plane jets of an ideal incompressible liquid has been regarded for several decades as not having an unambiguous solution (see [1-3], for example). The reason for the ambiguity is the mathematical indeterminacy of the problem, although it could be expected that with given values of the width of the colliding streams and the angle of impact, the configuration of the flow should be unambiguously determined. The interest in this problem stems from the fact that it is widely used to describe (in a first approximation) the high-speed oblique collision of metal plates.

Figure 1 shows the flow pattern in the collision of plane free jets having the same density $\rho = 1$ and velocity $v = 1$ (at an infinitely distant point and on the free boundaries). Here, a_1 and a_3 are the widths of the colliding jets (at the infinitely distant point), θ_3 is the angle of impact, a_2 and a_4 are the widths of the outgoing jets, and θ_2 and θ_4 are the angles of their inclination to the x axis (for simplicity, the angle θ_4 has been reduced by π). Three equations follow from the conditions of mass conservation for the flow and projections of the momentum flow. With allowance for the adopted notation, these equations take the form

$$a_1 + a_3 = a_2 + a_4,\ a_1 + a_3 \cos \theta_3 = a_2 \cos \theta_2 - a_4 \cos \theta_4, \qquad a_3 \sin \theta_3 = a_2 \sin \theta_2 - a_4 \sin \theta_4. \tag{1}$$

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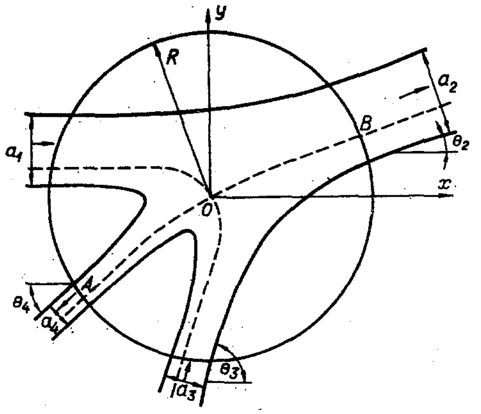

Fig. 1

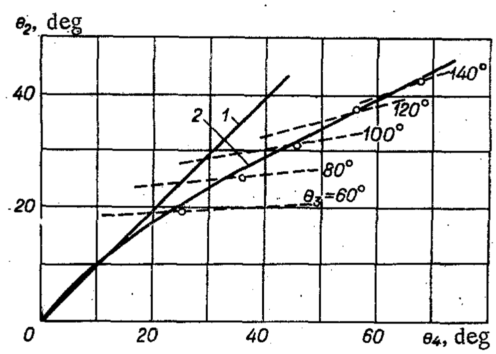

Fig. 2

Thus, we have only three equations to determine four unknowns (a_2, a_4, θ_2, and θ_4). We obtain the following relationship between the values of the angles θ_2 and θ_4 (the forward and reverse outgoing jets, respectively) from the three equations:

$$\theta_2 = 2\,\mathrm{arctg}\,[(\sin\theta_4 + m)/(\cos\theta_4 + n)] - \theta_4, \tag{2}$$

where $m = k\sin\theta_3/(1+k)$; $n = (1 + k\cos\theta_3)/(1+k)$; $k = a_3/a_1$; meanwhile, the permissible angles of inclination of the reverse jet are located within the range $0 \leqslant \theta_4 \leqslant \theta_3$. This remains valid for the case of collision of jets of the same width ($k = 1$) if no symmetry condition is imposed beforehand on the resulting flow configuration.

Thus, the exact solution [1-3] of the problem depends on one undetermined parameter (such as θ_4). Selection of a valud for this parameter requires that other considerations being taken into account. One of the best-known attempts to close the problem is the hypothesis in [4], from which it follows that the forward and reverse outgoing jets move in opposite directions at the limit ($\theta_2 = \theta_4$). Analysis of initial system (1) shows that such a value of the angles of the outgoing jets corresponds to the point of inflection on curve (2), while the reverse jet has the least possible width in this case (the flows of mass, momentum, and energy carried off by the reverse jet are minimal). The proposal that the forward and reverse outgoing jets move in opposite directions in the case of oblique collisions was made in [5] as well, but the arguments made in defense of this notion are not altogether convincing.

The ambiguity of the solution to the problem of the nonsymmetric collision of plane jets is sometimes interpreted as proof of the equivalence of all possible configurations and their instability [3]. Milne-Thompson reasoned that the flow configuration is determined by the initial phase of the collision, and that different initial conditions of the collision (such as a delay in the starting of one of the jets) "will correspond to different steady-state motions, although there are no grounds to presume that all of them will be stable" [1].

The possibility of evaluating the validity of a given hypothesis and argument has always been limited by the lack of reliable experimental data. In connection with this, an experimental study was made of the nonsymmetric collision of plane jets of liquid on a model unit [6]. The unit was shown to produce highly reliable results in sample jet problems. The modeling was done with several values of the ratio k of the widths of the colliding jets and selective values of collision velocity. We used a nozzle which produced an outgoing jet 20 and 30 mm wide; nonsymmetric collision was realized from symmetric conditions by cutting off part of the jet [6]. The discharge velocity ranged from about 2 to 4 m/sec.

The modeling of nonaxisymmetric collision showed that one steady-state flow configuration develops fairly stably for fixed values of k and θ_3, regardless of the initial conditions (delay in starting of one of the jets) or the width and velocity of the jet. We made several independent measurements of the angles of the outgoing jets both at different stages of one test and in different tests. The deviation of the measured values from their mean was no more than 1-1.5° for the angle θ_2 and 3-4° for the angle θ_4. If we change the direction of the outgoing jets by introducing some kind of interference, the same configuration is established after the interference is removed. This is evidence of the stability of the motion.

Thus, the completed experiment did not support the qualitative evidence in [1, 3] regarding the character of the jet flow that develops. Nevertheless, in the given problem there is a certain tendency for the reverse outgoing flow to become unstable. This is indicated by the large scatter of the experimental results for θ_4. It follows from Eq. (2) that this tendency should be manifest in a reduction in the angles of collision of the jets. For example, calculation by (2) shows that with $k = 0.5$ a change in the angle θ_2 by 1° changes θ_4 by 3-5° for angles of collision θ_3 = 120-100°, by 9-11° for θ_3 = 80-70°, and by 16-22° for θ_3 = 60-50°. Thus, it is understood that any of the distortions and irregularities always present in an experiment will have a greater effect on the reverse jet, the smaller the angle of collision. This is partly explained by the fact that the boundary of the reverse jet is less distinct than the boundary of the other jets in experiments. This situation is most obvious for angles of collision $\theta_3 \leqslant 100°$ and makes modeling practically impossible with $\theta_3 < 60°$ [6]. A similar result is obtained for cumulative jets. The problem being examined here is an approximate model for the formation of these jets. Experiments show that the instability of cumulative jets, leading to their distortion and even their breakdown into individual particles in the transverse direction, increases with a decrease in the angles of collision of plates or the angles of collapse of axisymmetric facings.

Figure 2 ($k = 1/2$) shows some results for the angles of the outgoing jets. Each angle of collision of the jets θ_3 can be used in conjunction with (2) to construct the curve of possible angles of inclination of the outgoing jets in the plane (θ_4, θ_2). Individual sections of these curves for several values of θ_3 are shown by the dashed lines. In accordance with the hypothesis in [4], the intersection of these curves at equal angles by straight line 1 gives values of the angles of the outgoing jets for the angle of collision corresponding to each curve. It is evident from Fig. 2 that the experimental values the angles θ_2 and θ_4 represented by the points lie well on the corresponding curves (2) and deviate greatly from line 1. The latter fact means that the Palatini hypothesis is invalid in the general case. Results similar to those in Fig. 2 were obtained for other values of k.

Figure 3 shows experimental data for the angles θ_4 (upper points) and θ_2 (lower points) of inclination of the outgoing jets in relation to the angle of collision of the jets for the case $k = 0.667$. The inequality of the angles of inclination of the forward and reverse outgoing jets is also apparent from this figure, although the difference between the angles is appreciably smaller than in Fig. 2 because this collision regime is closer to being symmetrical. Extrapolation of the experimental results (Figs. 2 and 3) to smaller collision angles suggests that for $\theta_3 \leqslant 50°$ the outgoing jets are nearly oppositely-directed.

The experimental modeling of jet collisions produced one more interesting result. Let a certain flow configuration (see Fig. 1) be realized in the collision of jets. We fix the direction of the outgoing jets and reverse the velocity vector in them, i.e., we make them collide; here, the two other jets become outgoing jets. Then the following question arises: Does the flow configuration remain the same? It turns out that in the general case it does not, i.e., the jets that became outgoing jets "choose" other directions. This is illustrated in the nonsymmetric collision of oppositely-directed jets of different widths (as is known [2, 3], this problem has a unique solution and was examined in [6] as a test problem). It follows from the calculations and experiments that the branching flow line in colliding jets has a point of inflection (Fig. 4). Direct experiments for a reversed flow showed that the motion of the outgoing jets is quite different from being oppositely directed. Along with the quantitative difference in the angles of inclination of the outgoing jets, there is one more important difference. It is evident from the experiments that for oblique collisions it is generally impossible to obtain a flow configuration for which the branching flow line in the outgoing jets has a point of inflection — which should occur in this problem if the configuration were to remain the same.

For special cases of collisions of jets with planar or central symmetry, the flow configuration remains the same when the jets reverse direction. This has also been shown experimentally (for example, see Fig. 5 in [6]).

Since it is still not clear which additional considerations must be taken into account to close the problem of the nonsymmetric collision of jets, we will attempt to describe empirically and quantitatively the experimental results obtained in the present study. Let us again examine the flow pattern shown in Fig. 1 and draw a circle of radius R from the coordinate origin at the flow stagnation point. With a fixed angle θ_3 of jet collision and a given radius R, we choose that flow configuration (i.e., the values of the angles $\theta_2(\theta_3, R)$ and $\theta_4(\theta_3, R)$ for which the section AOB of the branching flow line in the outgoing jets which is bounded by the circle has the shortest length). Numerical calculations showed that in this case $\theta_2(\theta_3, R) \to \theta_2^*(\theta_3)$ and $\theta_4(\theta_3, R) \to \theta_4^*(\theta_3)$ at $R \to \infty$. Meanwhile, even at R/a_1 = 8-10, the result does not depend on R. The values θ_2^* and θ_4^* of the angles of inclination of the outgoing jets agree fairly well with the experimental results. This is particularly evident

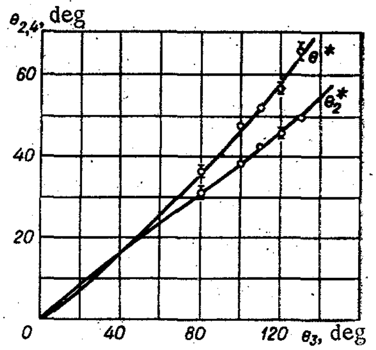

Fig. 3

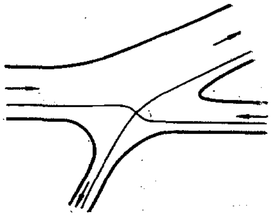

Fig. 4

from Figs. 2 (curve 2) and 3. Thus, together with experimental results, numerical modeling of the problem of the nonsymmetric collision of jets has made it possible to formulate the following hypothesis: In the oblique collision of plane jets of an ideal incompressible liquid, the flow configuration corresponds to the greatest length of the section of the branching flow line in the outgoing jets that is bounded by a circle of sufficient radius drawn from the flow stagnation point (an equivalent condition is that the curvature of this flow line be minimal).

This hypothesis of course empirical in nature and has no rigorous theoretical backing. There are no grounds for supposing that it will reliably describe all results of jet collisions. However, there are other positive aspects to the hypothesis apart from the good agreement with available empirical data for the angles of inclination of outgoing jets. Such agreement is obtained automatically in the oblique collision of jets of identical width without any a priori requirement on the symmetry of the resulting flow configuration. The hypothesis also describes the above experimental finding, indicating that in the general case the configuration of the jet flow changes when the direction of the flow is reversed.

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